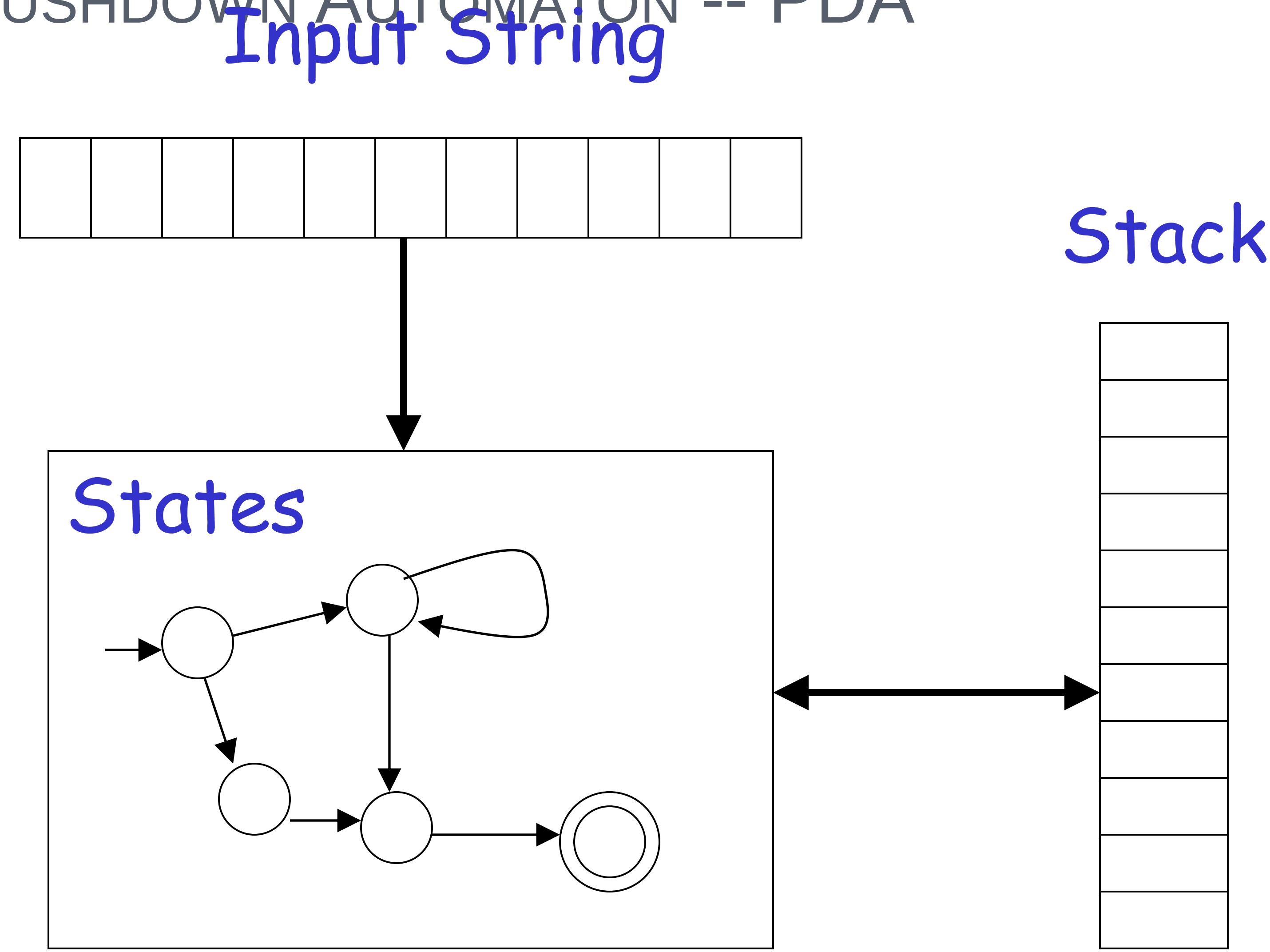
Pushdown Automata

Pushdown Automaton -- PDA

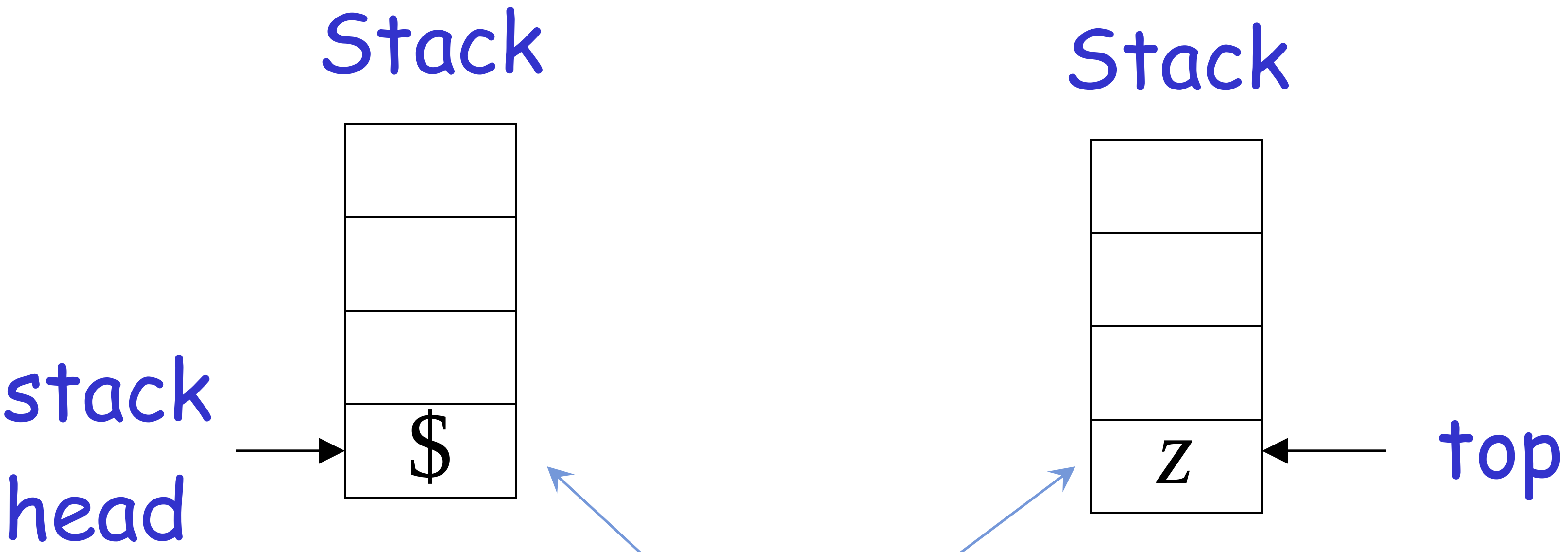


Input String

Stack

States

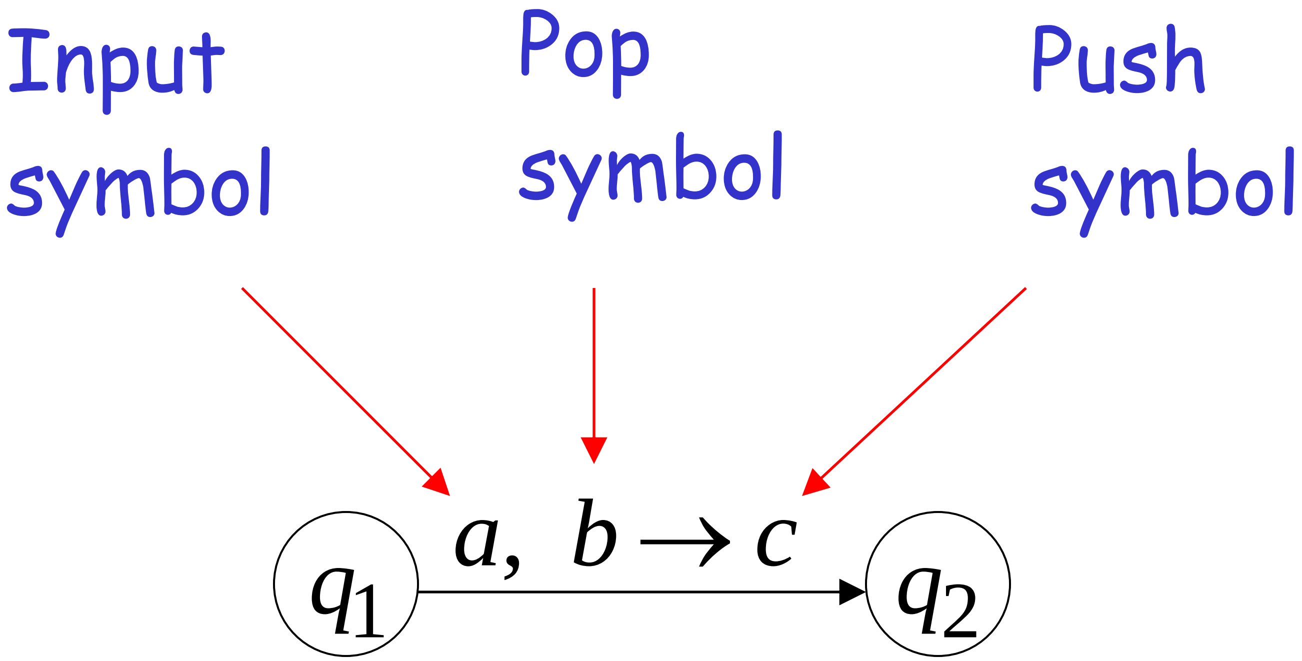
Initial Stack Symbol



bottom special symbol

Appears at time 0

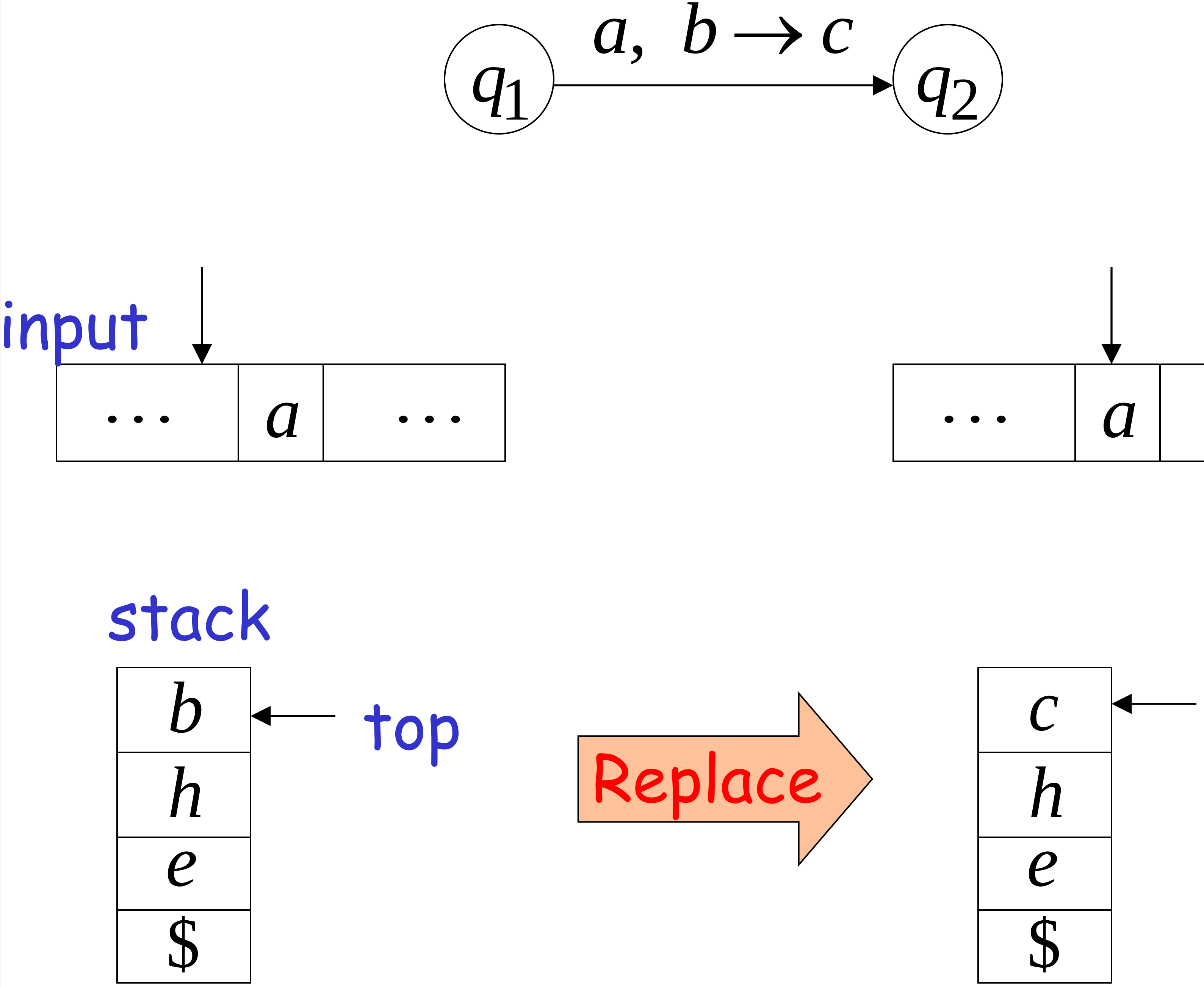
The States

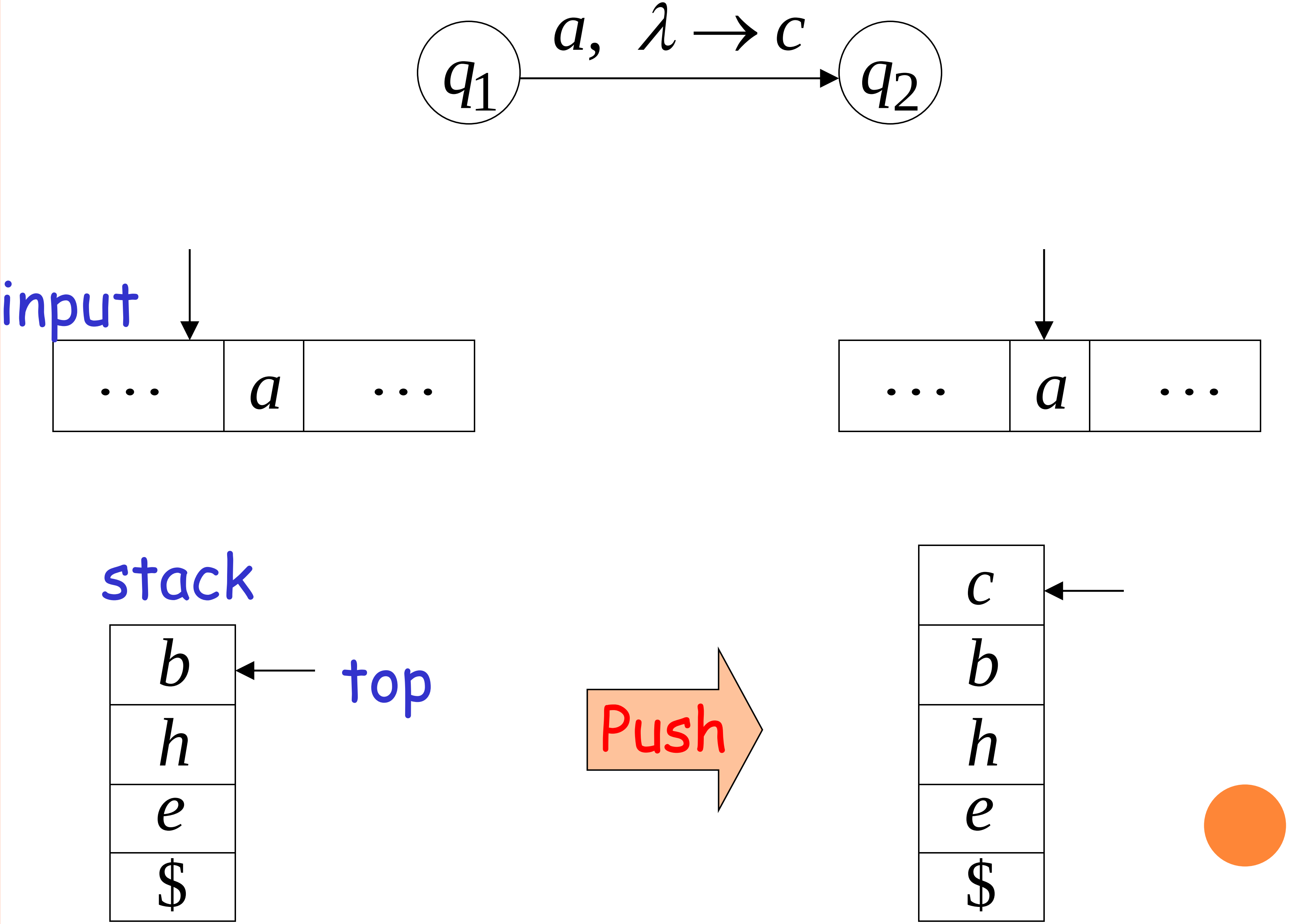


Input symbol

Pop symbol

Push symbol


$a, \; b \rightarrow c$
q_1
q_2
input
$\cdots$
a
$\cdots$
$\cdots$
a
$\cdots$
stack
top
b
h
e
$\$$
Replace
c
h
e
$\$$



q_1

$a, \lambda \rightarrow c$

q_2

input

$\cdots$ a $\cdots$

$\cdots$ a $\cdots$

stack

b

h

e

$\$$

top

Push

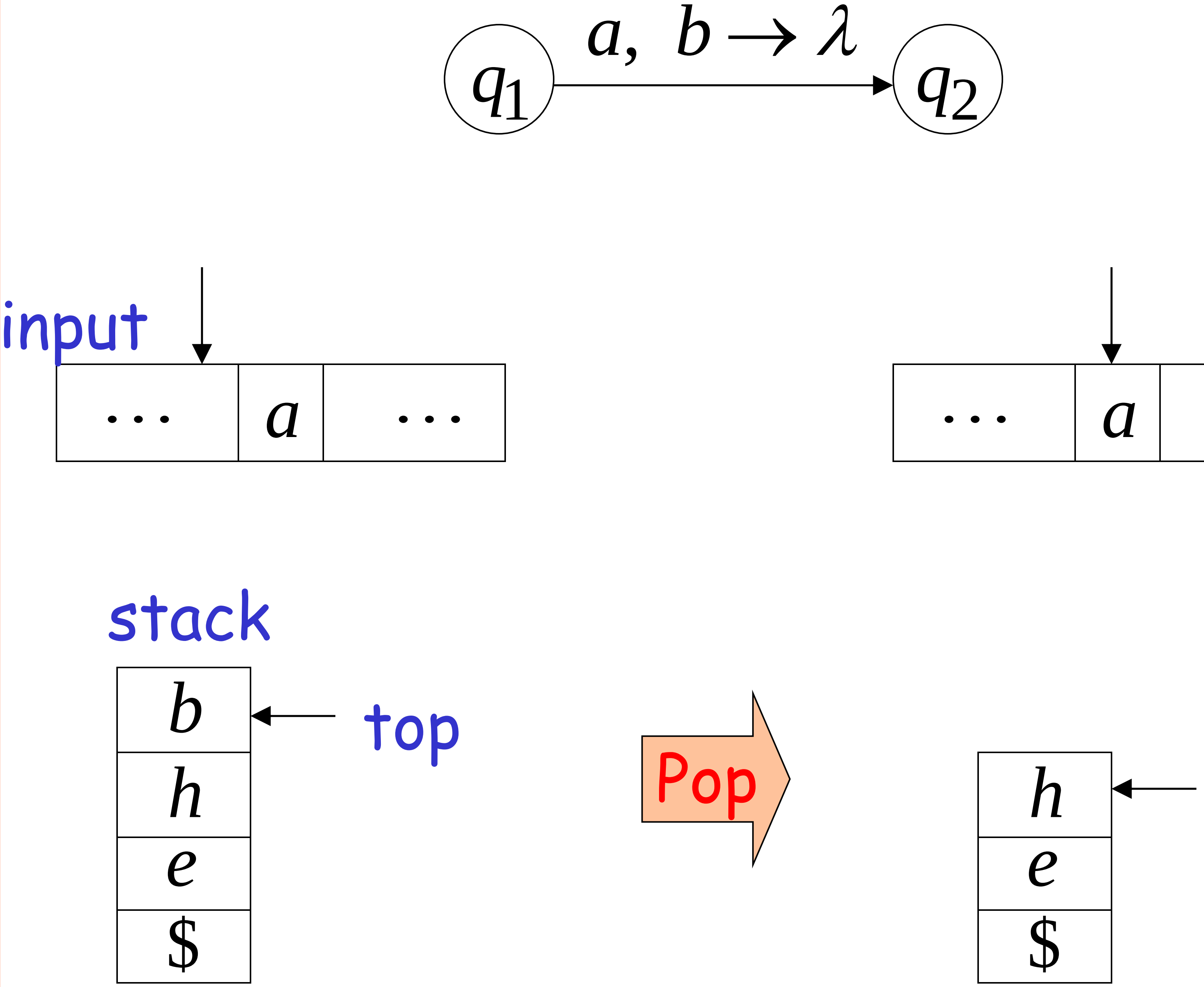
c

b

h

e

$\$$



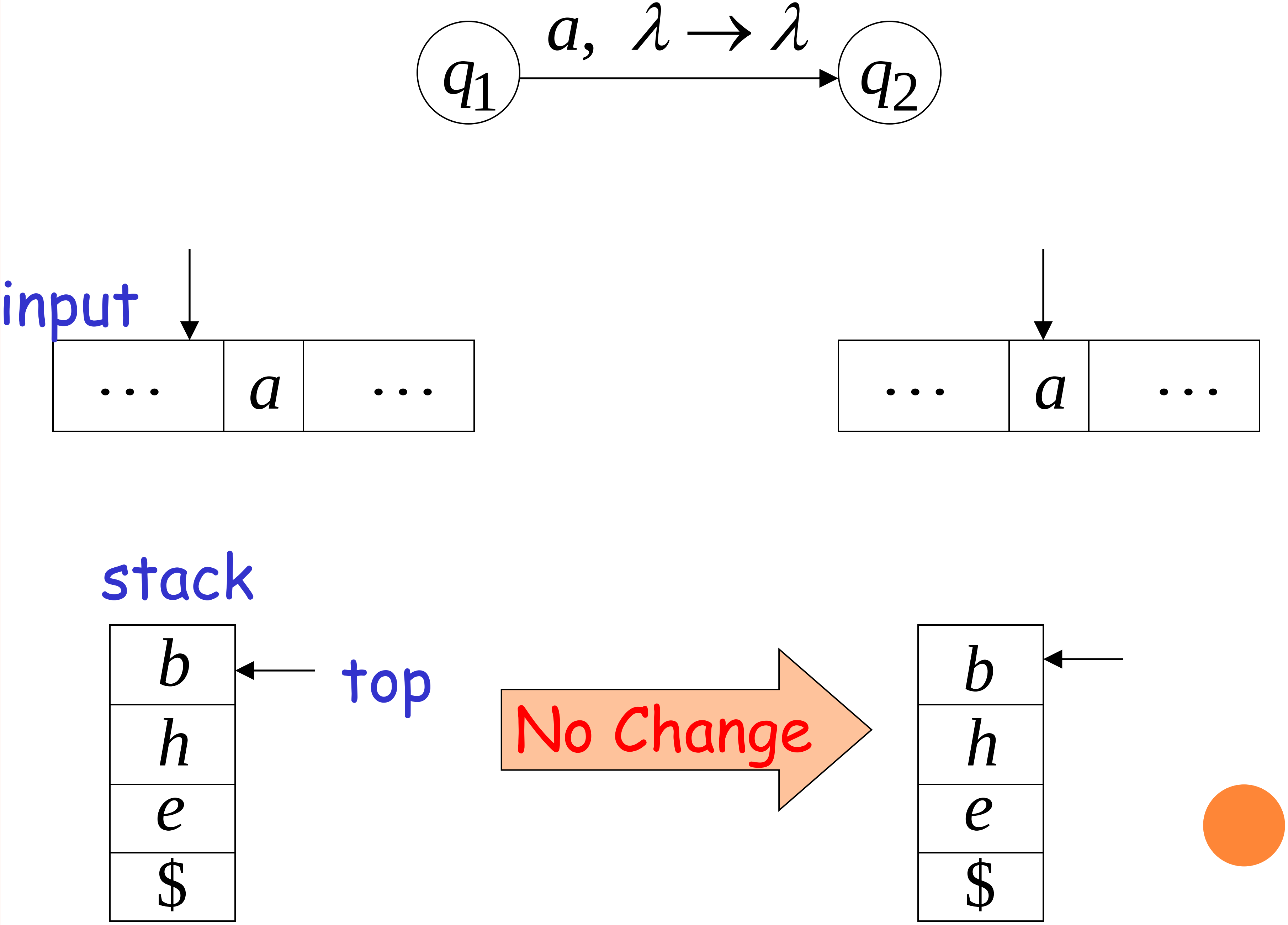
$q_1 \xrightarrow{a,\ b \to \lambda} q_2$

input

stack

top

Pop



$q_1 \xrightarrow{a,\ \lambda \to \lambda} q_2$

input

$\cdots$	a	$\cdots$

$\cdots$	a	$\cdots$

stack

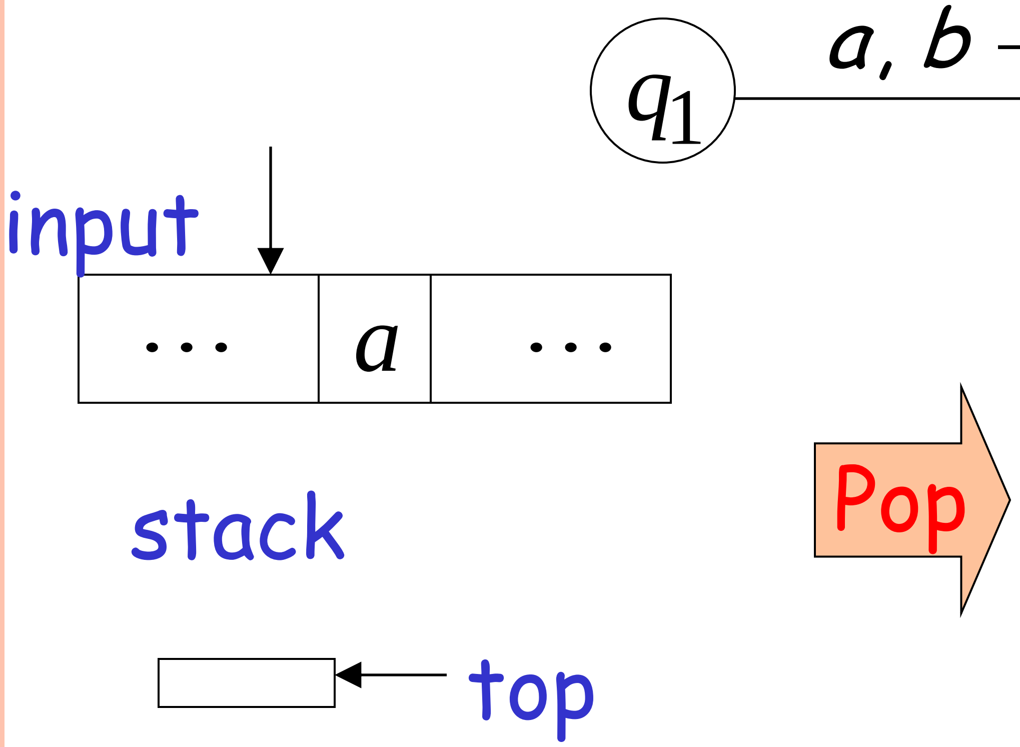
top

b
h
e
$\$$

No Change

b
h
e
$\$$

Pop from Empty Stack



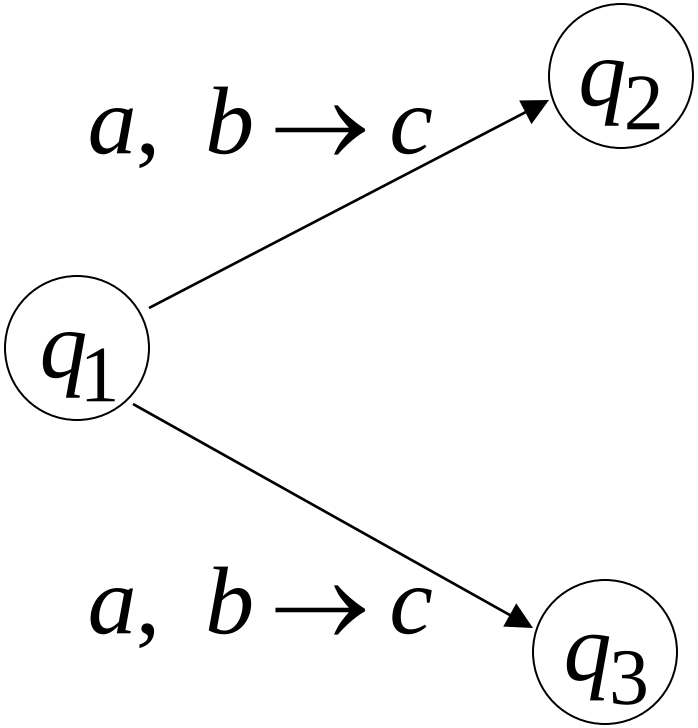
Automaton halts!

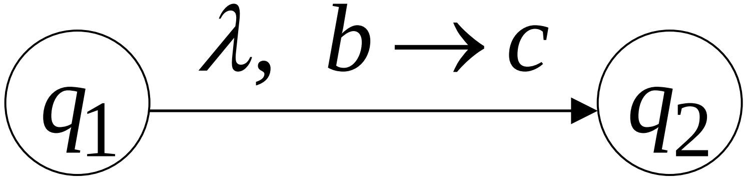
If the automaton attempts to pop from empty stack then it halts and rejects input

Non-Determinism

PDAs are non-deterministic

Allowed non-deterministic transitions



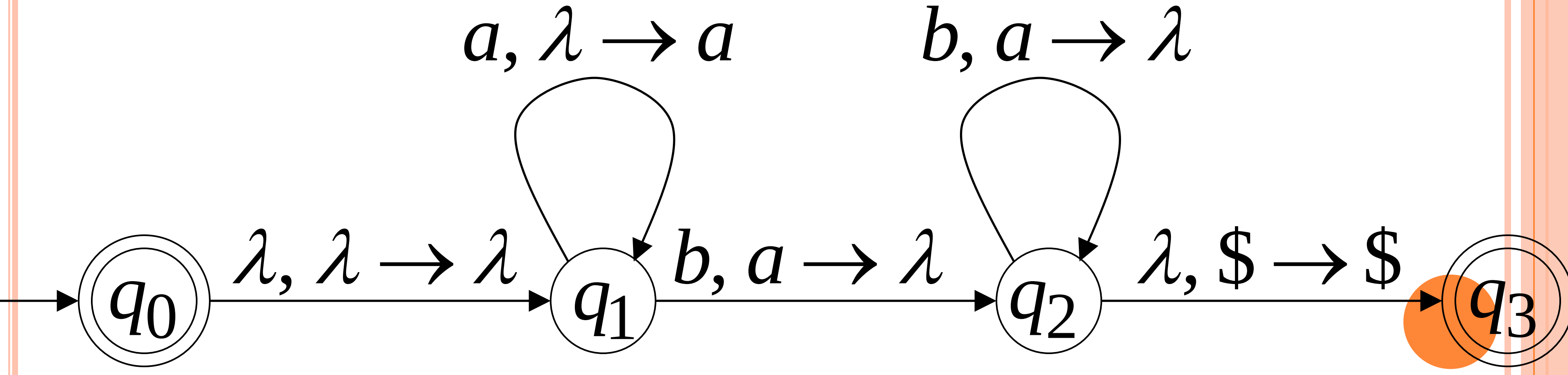


$\lambda -$ transition

Example PDA

PDA M:

$$L(M) = \{a^n b^n : n \geq 0\}$$


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
q_0
$\lambda, \lambda \rightarrow \lambda$
q_1
$b, a \rightarrow \lambda$
q_2
$\lambda, \$ \rightarrow \$$
q_3

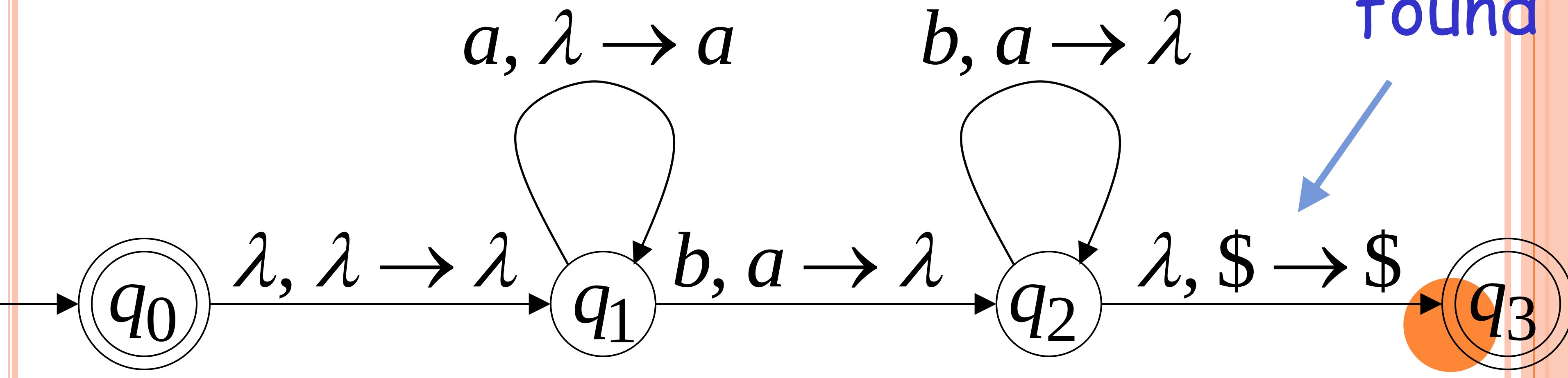
$$L(M) = \{a^n b^n : n \geq 0\}$$

Basic Idea:

1. Push the a's on the stack

2. Match the b's on input with a's on stack

3. Match found



Execution Example: Time 0

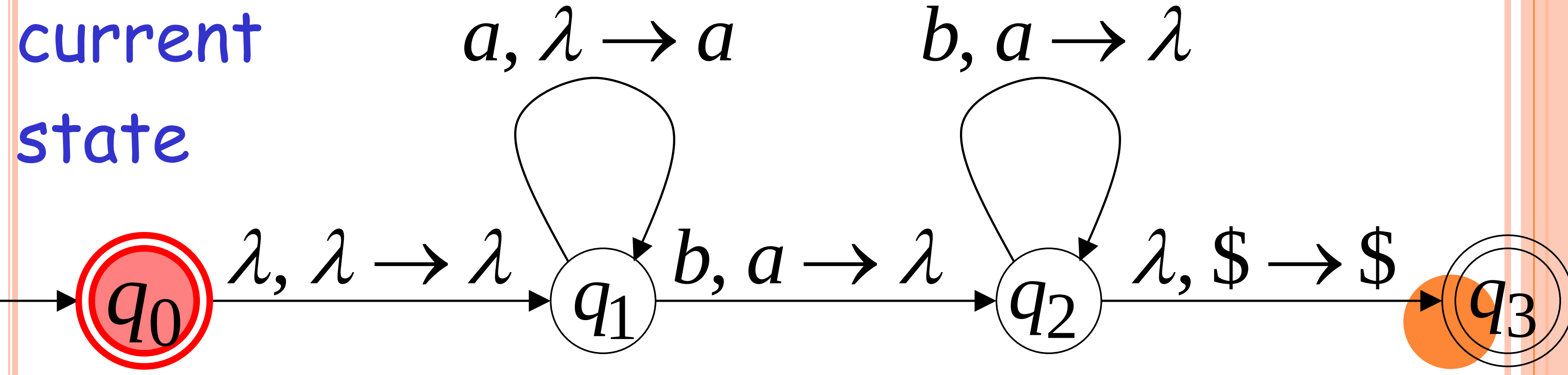
Input

a	a	a	b	b	b

$\$$

Stack

current state


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

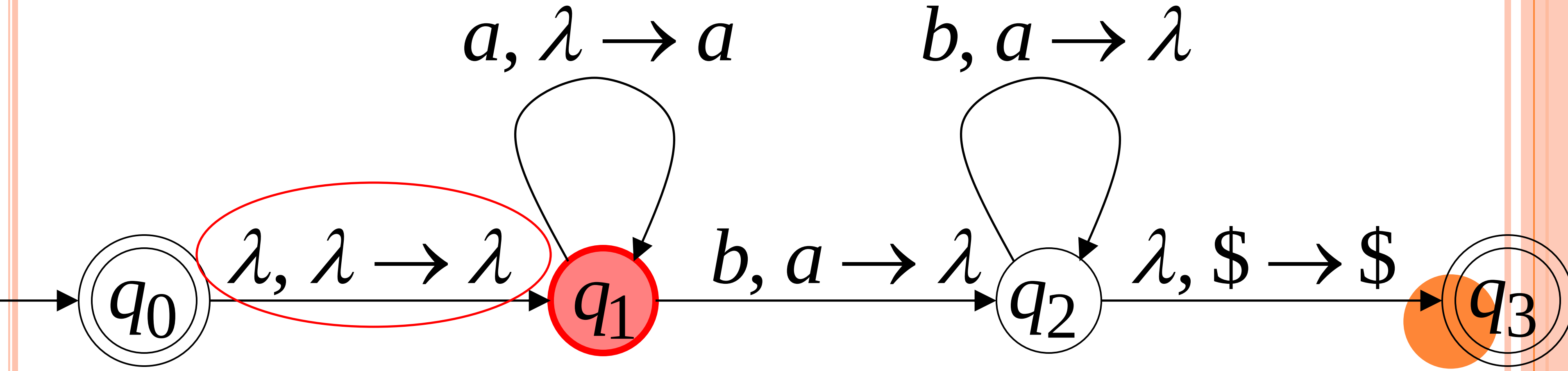
Time 1

Input

a	a	a	b	b	b

$\$$

Stack


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

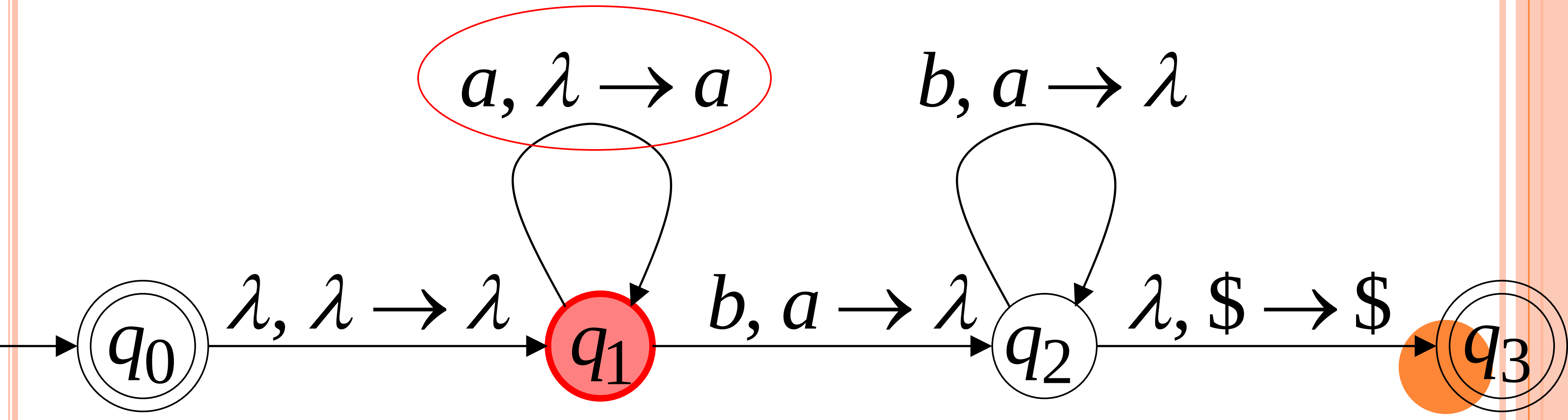
Time 2

Input

a	a	a	b	b	b

a
$\$$

Stack



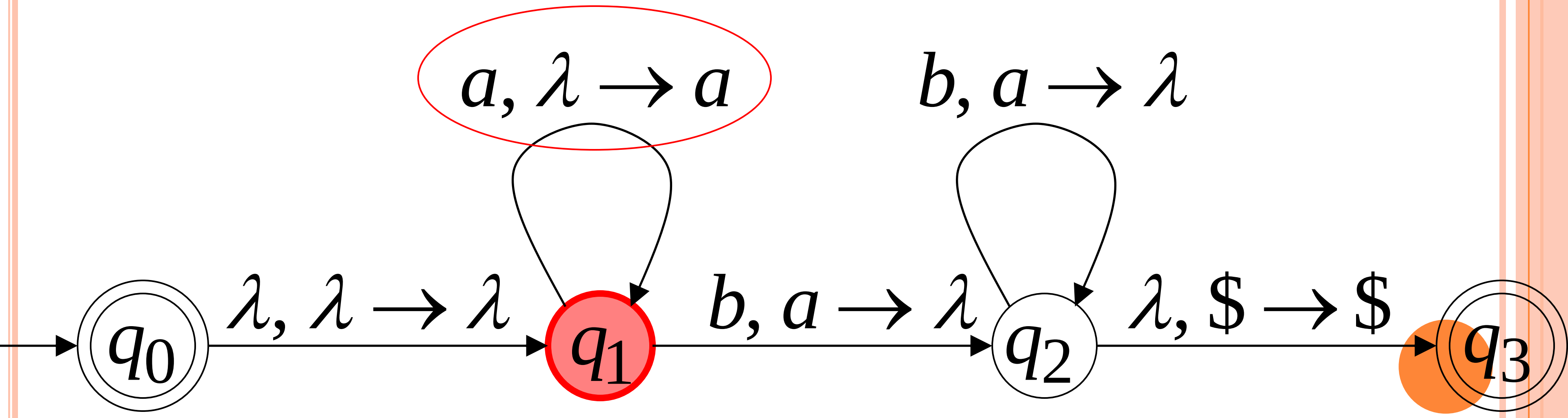
Time 3

Input

a	a	a	b	b	b

a
a
$\$$

Stack



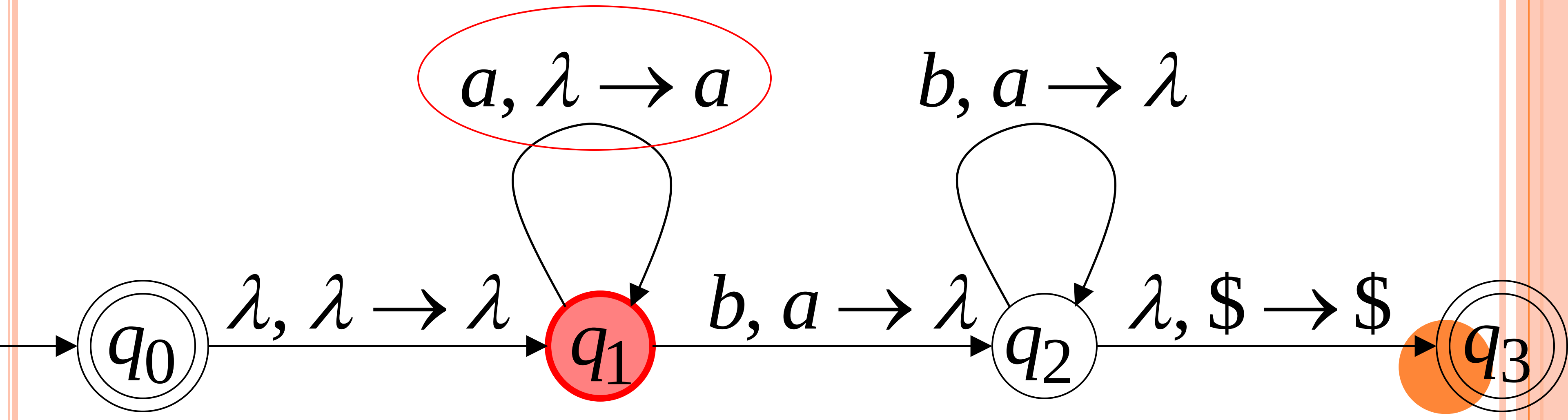
Time 4

Input

a	a	a	b	b	b

a
a
a
$\$$

Stack


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

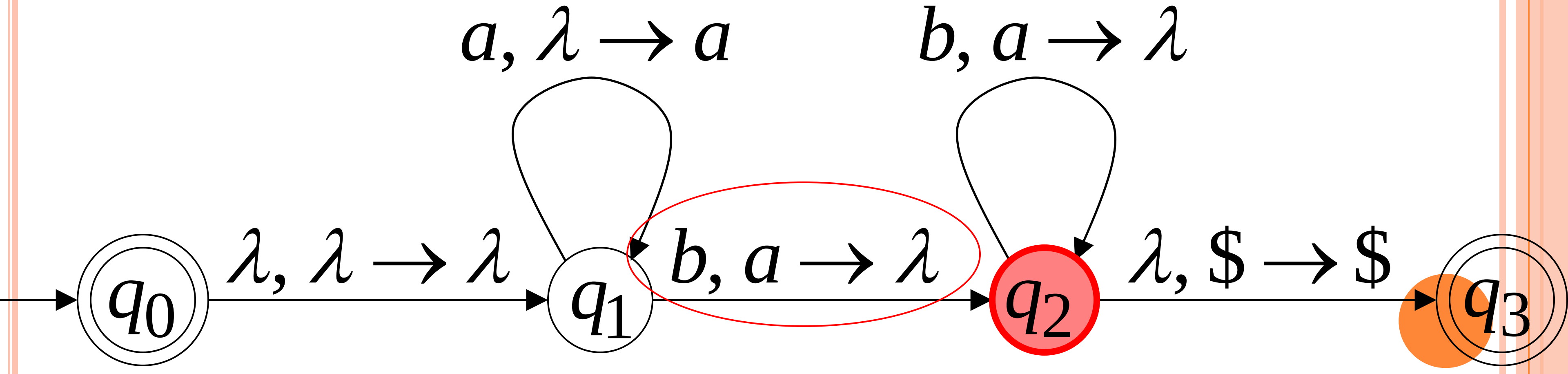
Time 5

Input

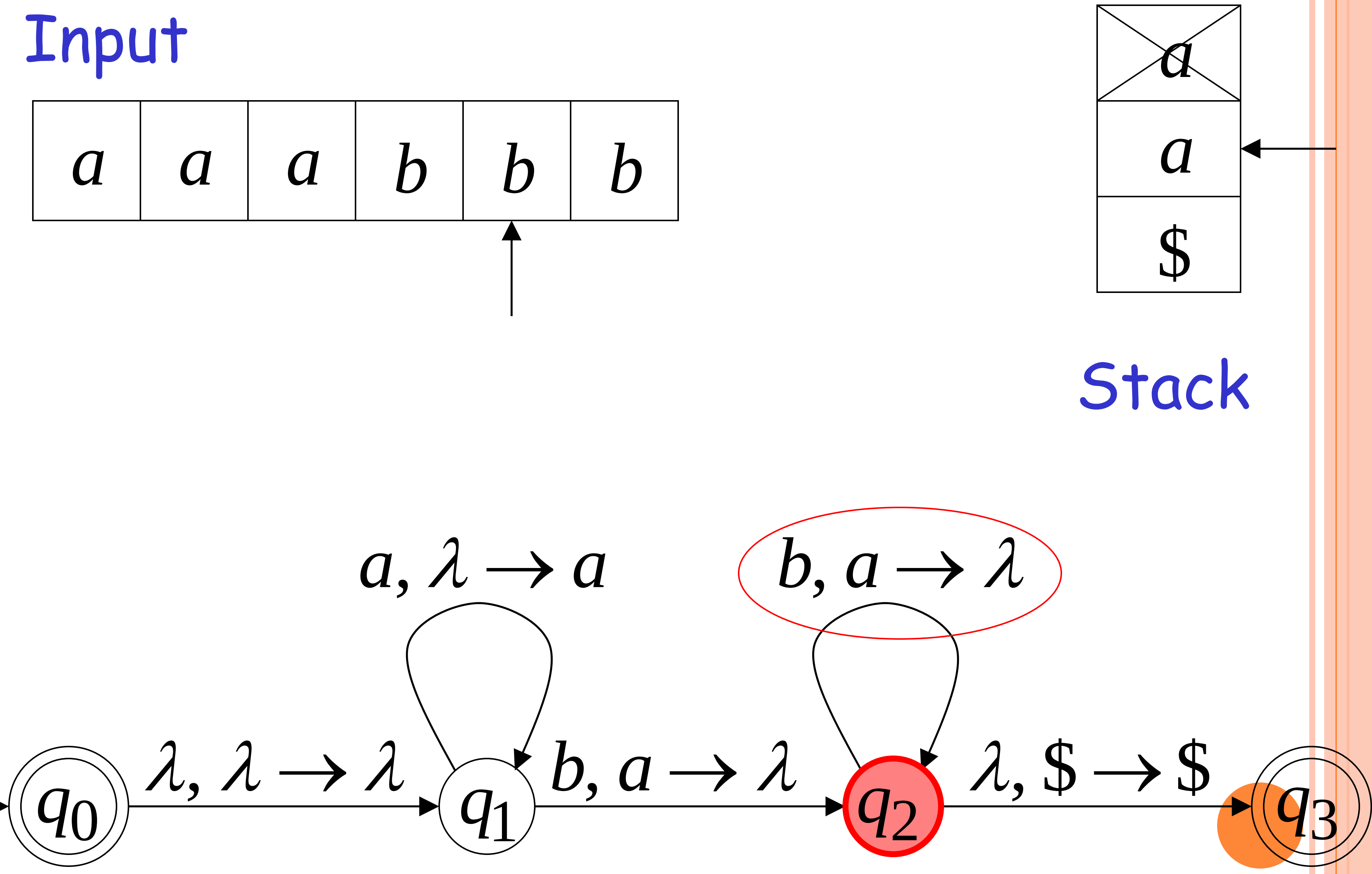
a	a	a	b	b	b

~~a~~
a
a
$\$$

Stack



Time 6


Input

Stack

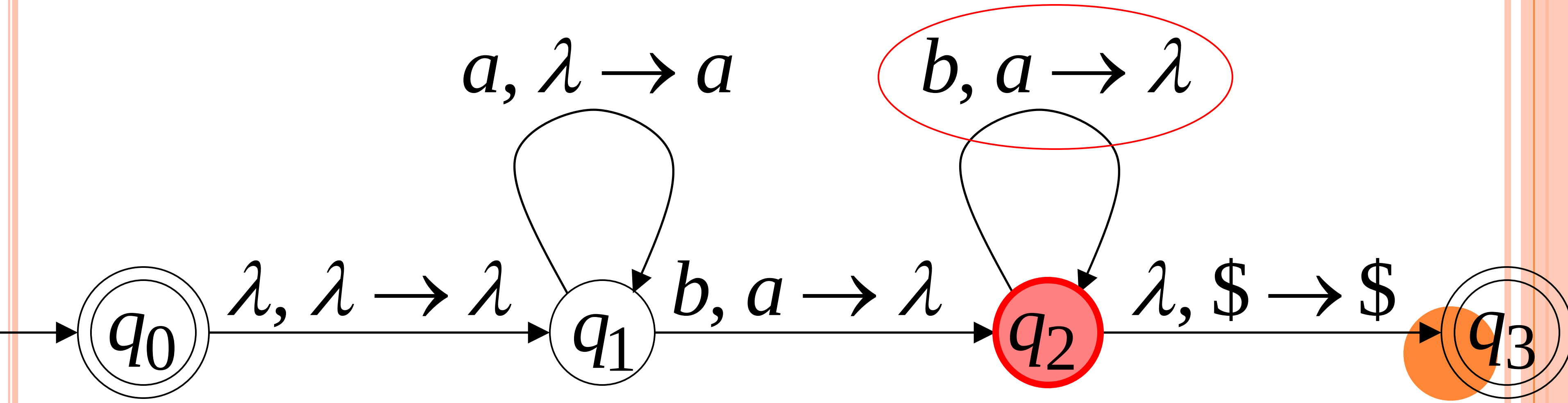
Time 7

Input

a	a	a	b	b	b

$\cancel{a}$
$\$$

Stack



$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

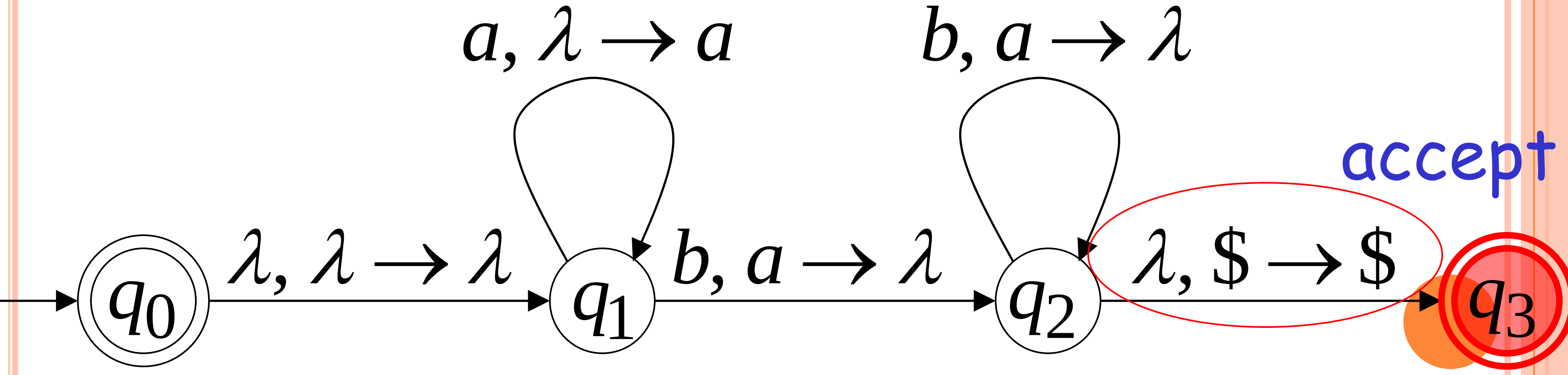
Time 8

Input

a	a	a	b	b	b

$\$$

Stack



A string is accepted if there is a computation such that:

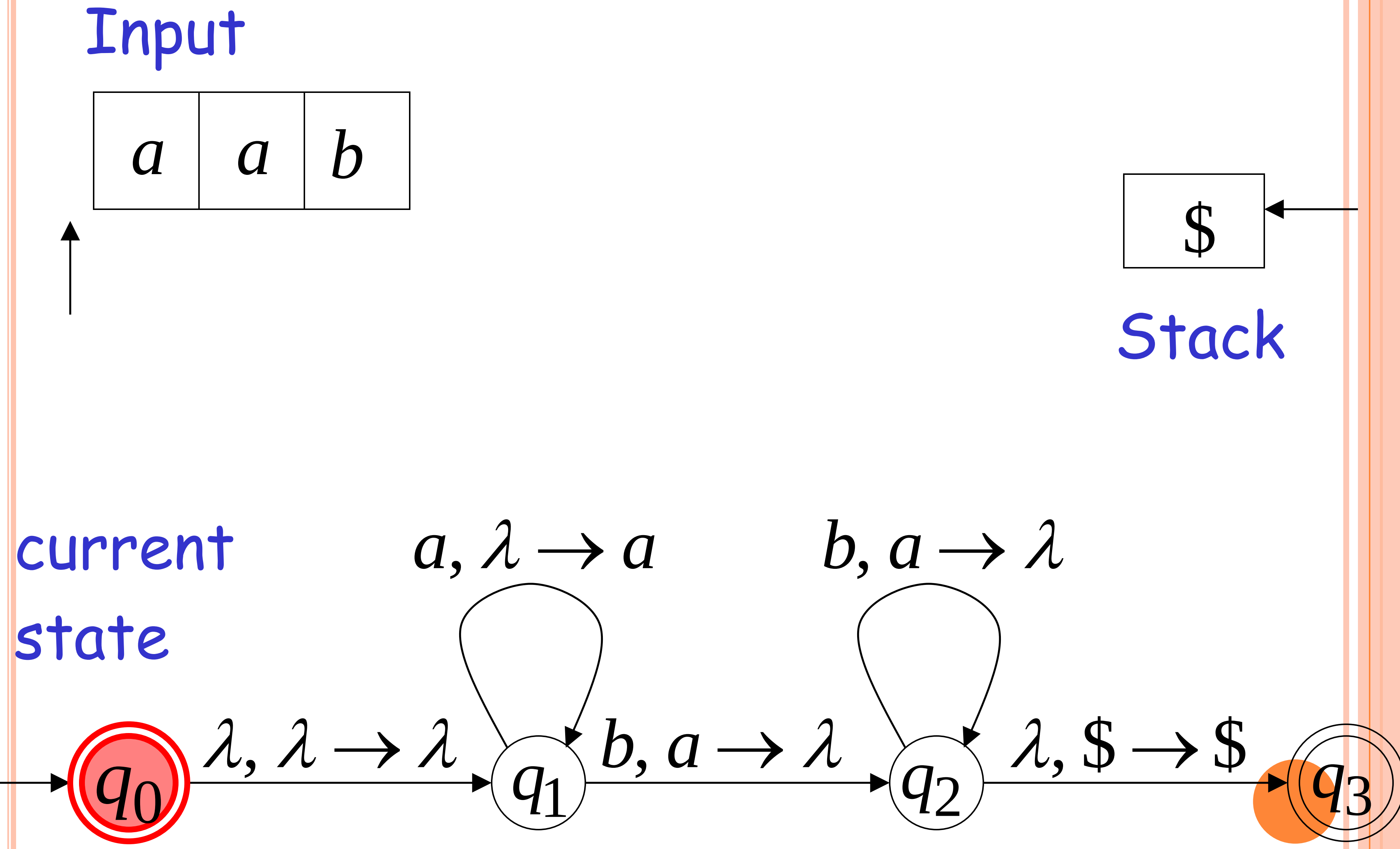
All the input is consumed

AND

The last state is an accepting state

we do not care about the stack contents at the end of the accepting computation

Rejection Example: Time 0



Input

Stack

current state

a
a
b
$\$$
$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

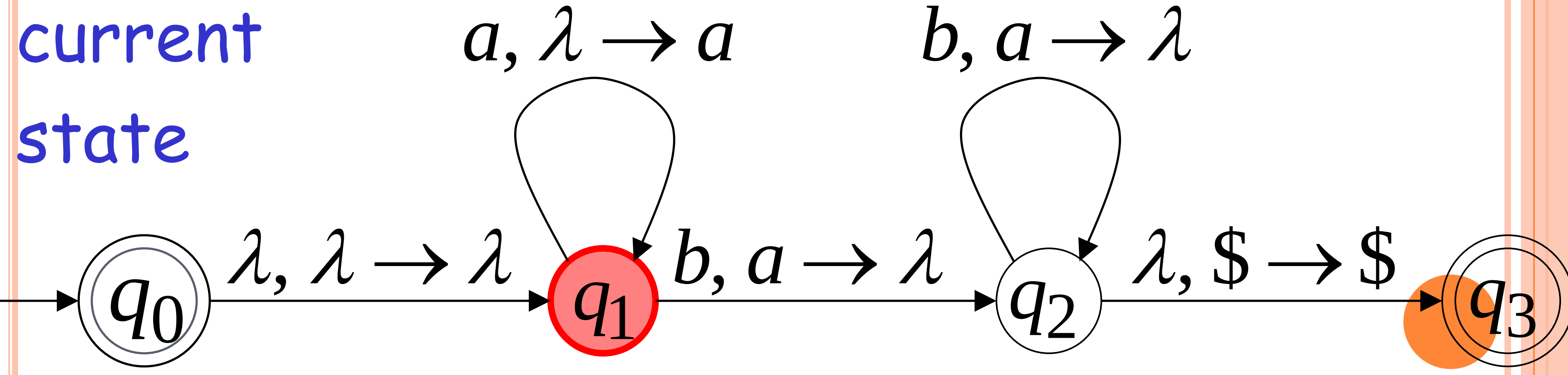
Rejection Example: Time 1

Input

a	a	b

$\$$

Stack



current state

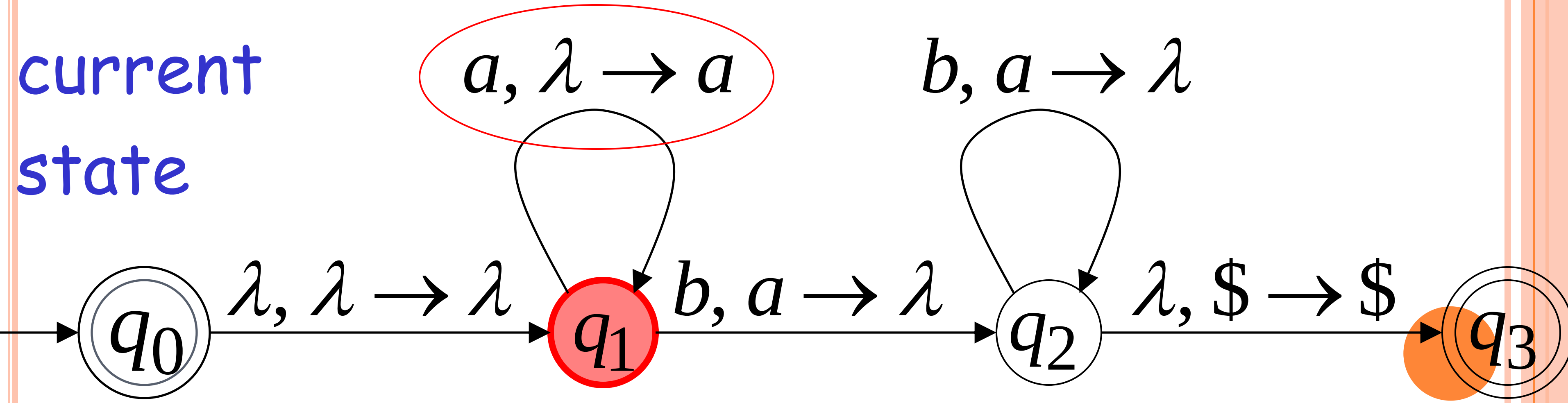
Rejection Example: Time 2

Input

a	a	b

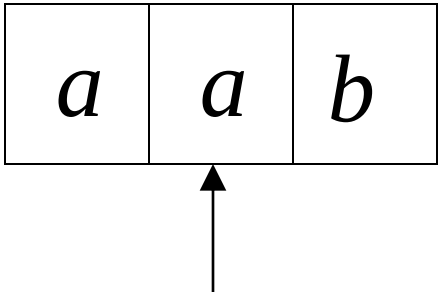
a
$\$$

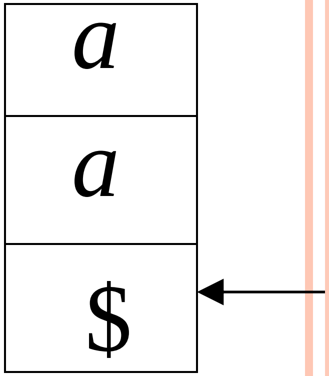
Stack


current
state
$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

Rejection Example: Time 3

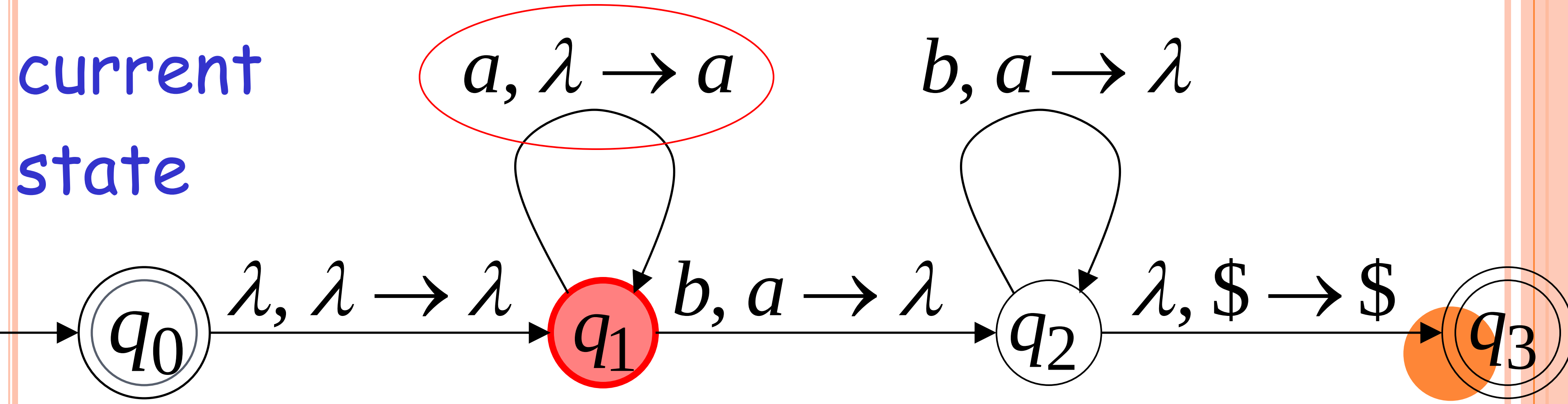
Input





Stack

current state


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

Rejection Example: Time 4

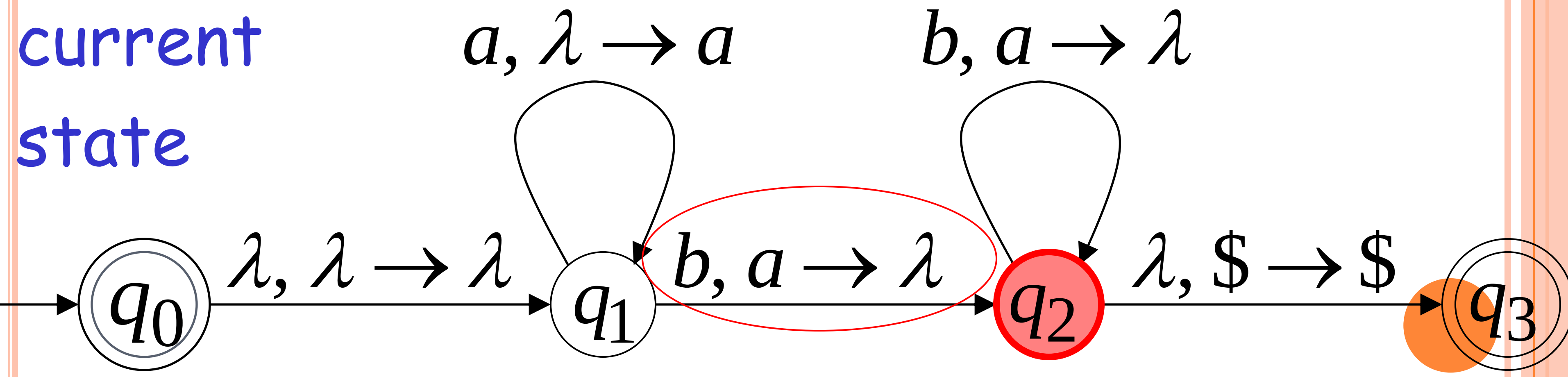
Input

a	a	b

$\cancel{a}$
a
$\$$

Stack

current state



$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

Rejection Example: Time 4

Input

a	a	b

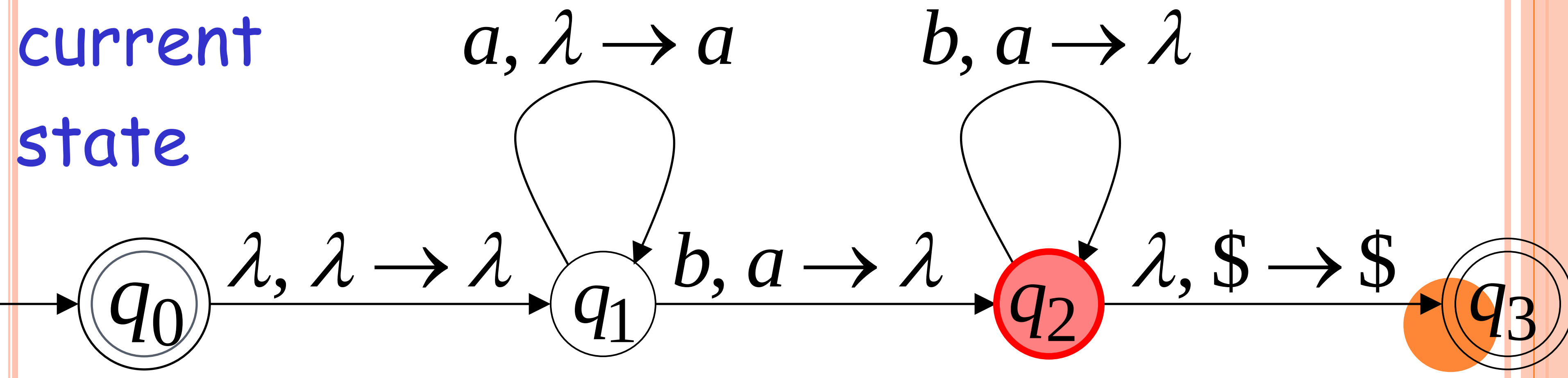
Stack

$\cancel{a}$

a

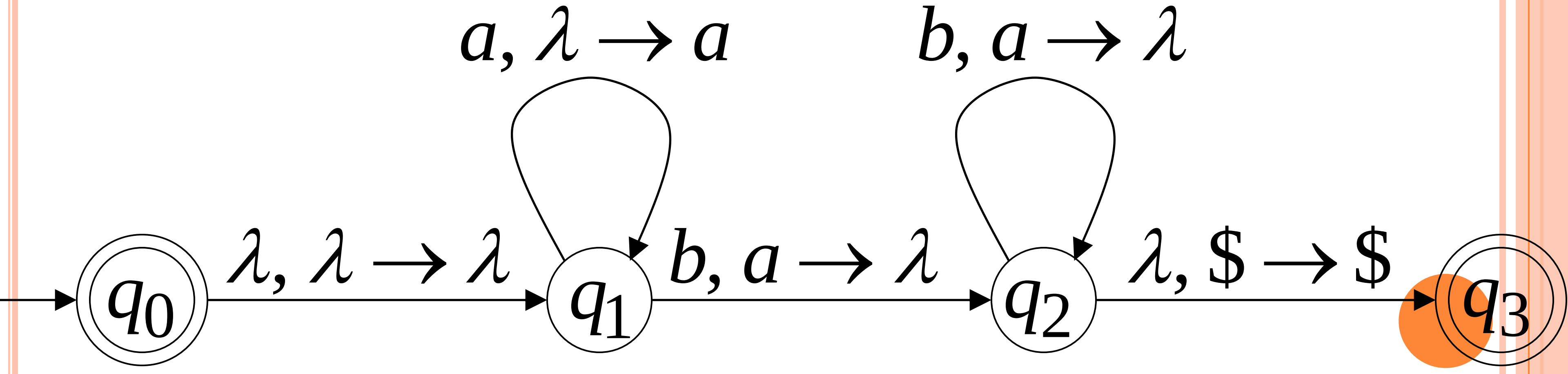
$\$$

reject


current state
$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3

There is no accepting computation for aab

The string aab is rejected by the PDA


$a, \lambda \rightarrow a$
$b, a \rightarrow \lambda$
$\lambda, \lambda \rightarrow \lambda$
$b, a \rightarrow \lambda$
$\lambda, \$ \rightarrow \$$
q_0
q_1
q_2
q_3